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FRACTURE CRITERION OF ISOTROPIC MATERIALS

BY PAO C. HUANG

RESEARCH AND TECHNOLOGY DEPARTMENT

SEPTEMBER 1986

NAVAL SURFACE WARFARE CENTER

Dahlgren, Virginia 22448-5000 • Silver Spring, Maryland 20903-5000

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FOREWORD

Fracture mechanics is becoming more and more important in structural analyses. In order to ensure the safety of a structure, the analyst must be able to predict under what stress state will a crack be initiated. Will it propagate? Will the structure have a catastrophic failure or can it still take its design load? If it can take the load, for how long? An attempt to answer these questions has been pursued, but in most fracture mechanics analyses, the crack initiation criteria is always left out. Most of these analyses start out with a crack or material flaw of some sort, then proceed to answer the remaining questions. The author saw this, and while performing structural analyses for different tasks at the Naval Surface Warfare Center, he conceived the idea of developing a semi-empirical fracture criterion for the determination of crack initiation in a multiaxial stress state.

This work was performed in the Metallic Materials Branch (Code R32) and has been reviewed by Dr. P. W. Hesse (R32 Head) and John P. Matra, Jr. (R32).

Approved by:

Dr. C. E. Mueller

DR. C. E. MUELLER, Head
Materials Division

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ILLUSTRATIONS

INTRODUCTION

To determine the initiation of a mechanical fracture in a weapon structure, fracture criterion in terms of multiaxial stress space is often required. Although information of the uniaxial ultimate stress is readily plentiful, it is not sufficient for such prediction in the biaxial and triaxial stress states. From the evidence in the experimental data, a material fractures complicatedly in multiaxial stress states. McAdam[1] found that the ultimate stress in triaxial stress state was approximately two times the value of that of the uniaxial for the same material. Plastic flow in the fracture surface was often not observed even in a ductile material, especially under blast load. Unlike the yielding of material, the fracture depended strongly on the mean stress at the breaking point. Since the Mises yield criterion is a right cylinder along the mean stress axis, then the fracture criterion could be a revolutionary surface such as a cone, paraboloid, or something else with its radius varying along the same mean stress axis. According to the work by McAdam,[1] the apex would be on the axis which represents a triaxial stress state and equals twice the uniaxial value.

Based on the foregoing phenomena, a cone or a paraboloid may be a permissible fracture surface in a principal stress space. Its apex will be on the mean stress axis which has equal direction cosines with the coordinate axes, and the intercepts on the principal axes represent the values of the uniaxial ultimate stress at proper strain rate.

The simplest form of the surfaces of revolution is a right cone. The next higher order of the surface is a paraboloid. Both yield simple criteria for isotropic materials. Without elaborate testing, one may be preferred over the other for better correlation.

These fracture criteria in the form of surfaces of revolution show the following properties:

1. uniaxial ultimate stress is employed at the intercepts of the principal axes

2. the apex has an ultimate stress equal to twice the uniaxial value, it is in an equal triaxial stress state

3. ultimate stress in the biaxial stress state represented by the surface of revolution

4. criterion is dependent on the mean stress

5. proper strain rate data can be used if necessary.

STRESSES IN THE PRINCIPAL STRESS SPACE

Figure 1 shows the geometrical representation of stresses in principal stress space. The coordinate system is a right hand triad in terms of principal stresses in the 1, 2, and 3-directions. Any point P is a stress tensor with principal stresses S_1, S_2, and S_3. A mean stress axis, n-axis, can be drawn from the origin outward having an equal angle with each principal axis. The direction cosine of this angle is 0.58. Any point on the n-axis has three equal components, S_m, along the three principal stress axes. S_m is exactly the mean stress of the stress tensor. The deviatorial stress components D_i

$$D_i = S_i - S_m \qquad \text{where } i = 1 \text{ to } 3$$

are the components of a vector $\underline{NP}$ which is normal to the n-axis. Therefore, any stress vector $\underline{OP}$ can be decomposed into two vectorial components $\underline{ON}$ and $\underline{NP}$. The plane in which the vector $\underline{NP}$ lies is called the deviatorial plane. It has an importance in the determination of fracture initiation because it gives the deviatorial stress of the stress tensor and also the fracture criterion at the same point.

GEOMETRICAL REPRESENTATION OF FRACTURE AND YIELD CRITERIA

Figure 2 shows a geometrical representation of a fracture cone and the Mises yield criteria. Mises yield criterion is a prismatic cylinder along the mean stress axis. Being a cylinder of constant radius, f,

$$f = (\sqrt{(2/3)}) * Y \qquad \text{where Y is the yield stress of the material,}$$

the yield value is also constant and independent of the mean stress.

The fracture cone criterion having its apex, N, on the mean stress axis is dependent on the mean stress value. Its equation has the form

$$f = (\sqrt{6} * (2U - S_m))/5 \qquad \text{where U is the uniaxial ultimate stress.}$$

Notice that the function, f, for either yield or fracture criterion represents a circle in the deviatorial plane with its origin at the mean stress axis. The intercept of the cone with any principal axis has the value U, while the apex, N, has three components equal to 2U. Figure 3 shows a two-dimensional plot in the S_3-S_m plane of the Mises yield, fracture cone, and fracture paraboloid criteria. The paraboloid has an equation as follows:

$$f = \sqrt{.4U(2U - S_m)}$$

It can be seen that the equation of the paraboloid is still rather tractable for a higher order geometric surface. The selection of either the cone or the paraboloid should be determined, if feasible, by a few proper biaxial tests. It is interesting to note that beyond a certain point on the mean axis the radius of the yield cylinder is

actually greater than that of the fracture surface. It indicates that fracture of the material can occur before plastic flow starts. This phenomenon has often been observed in the tests of mild steel cylinders, especially under blast loading.

DERIVATION OF FRACTURE CRITERIA

For an isotropic material, the fracture cone or the paraboloid is a body of revolution about the mean stress axis.

Let

U_1 = uniaxial ultimate stress
U_3 = triaxial ultimate stress
S_m = mean stress of the stress tensor

then the cone criterion, f_c, has the following form

$$f_c = ((\sqrt{6}(U_3 - S_m))/(3U_3 - U_1)$$

and the paraboloid criterion, f_p, has

$$f_p = U_1 * (\sqrt{2(U_3 - S_m)/(3U_3 - U_1)})$$

In these general expressions the value of U_3 is experimentally difficult to obtain, therefore, the result by McAdam[1] will be used for the following development until a better hypothesis can be established.

For $U_1 = U$ and $U_3 = 2U$, then

$$f_c = (\sqrt{6} * (2U - S_m))/5$$

and

$$f_p = \sqrt{.4U(2U - S_m)}$$

at the intercept at the principal stress axis. Where $S_m = U/3$, the radius of either surface is

$$f_c = f_p = (\sqrt{2/3}) * U$$

which yields the principal stresses

$$S_1 = S_2 = S_3 = U$$

at the apex of the surface. Where $S_m = 2U$, therefore, f equals zero for either case.

Since f is the radius of a circle in the deviatorial plane, therefore, the equation of the circle is

$$\underline{R} = f\,\underline{r}$$

where $\underline{r}$ is a unit vector in the plane with its origin at the mean stress axis. Now the stress tensor has a deviatorial component $\underline{NP}$ in the form of

$$\underline{NP} = D\underline{r}$$

To initiate fracture, $\underline{NP}$ must be equal to or greater than $\underline{R}$. Hence

$$D = \text{or} > f$$

Using the second invariant of the deviatorial stress tensor, J_2, the criterion become

$$J_2 = \text{or} > (f)^2/2$$

since $D^2 = 2J_2$.

Therefore, for the cone one has

$$J_2 = \text{or} > (3(2U - S_m)^2)/25$$

for the paraboloid one has

$$J_2 = \text{or} > (U(2U - S_m))/5$$

Another popular parameter, the effective stress, S_e, has been widely employed in the theory of plasticity. Therefore, it also becomes feasible to use this parameter in the fracture criterion.

Since

$$(S_e)^2 = 1.5 * D^2 = 3J_2$$

therefore, for a cone one has

$$S_e = \text{or} > .6 * (2U - S_m)$$

and, for a paraboloid one has

$$S_e = \text{or} > \sqrt{.6U(2U - S_m)}$$

STRAIN RATE EFFECTS

It is well known that both the yield stress and the ultimate stress would in general increase with strain rate. For most materials, the yield stress would grow faster than the ultimate stress. M. J. Manjoine[2] has measured the properties of a mild steel at a wide range of strain rate; the data are presented in the following chart.

STRAIN RATE	.000001	.00001	.0001	.001	.01	.1	1	10	100	1000
Y/U IN (%)	51	54	57	60	61	66	73	83	92	98

It can be seen that yield/ultimate ratio is increasing steadily. This means that the possibility of having appreciable plastic flow in the material before fracture is diminishing. Based on these values, three sets of the yield and the fracture criteria are calculated and plotted in a biaxial principal stress space as shown in Figure 4. The phenomenon of decreasing in ductility due to high strain rate is clearly evident.

In a series of tests to determine the vulnerability of stiffened shells under blast loads, fracture was found in the web of a stiffening ring which exhibited little or no plastic flow in the cracked surfaces. Figure 5 shows the results of a computer run that the exact area of the web has exceeded the fracture criteria. In addition, the strain rate at that point is calculated approximately equal to 1000 in/in/sec. A rate at which brittle fracture always prevails.

CONCLUSION

A semiempirical fracture criterion has been presented here for the determination of crack initiation in a multiaxial stress state. The type of fracture surface may be selected with properly designed biaxial testing programs while the value of uniaxial ultimate strength can be obtained experimentally in proper strain rate and temperature environments. However, the next logical step seems to be the planning of an elaborate testing program aimed to evaluate the validity of this proposed criterion and to establish its applicability in practical engineering problems.

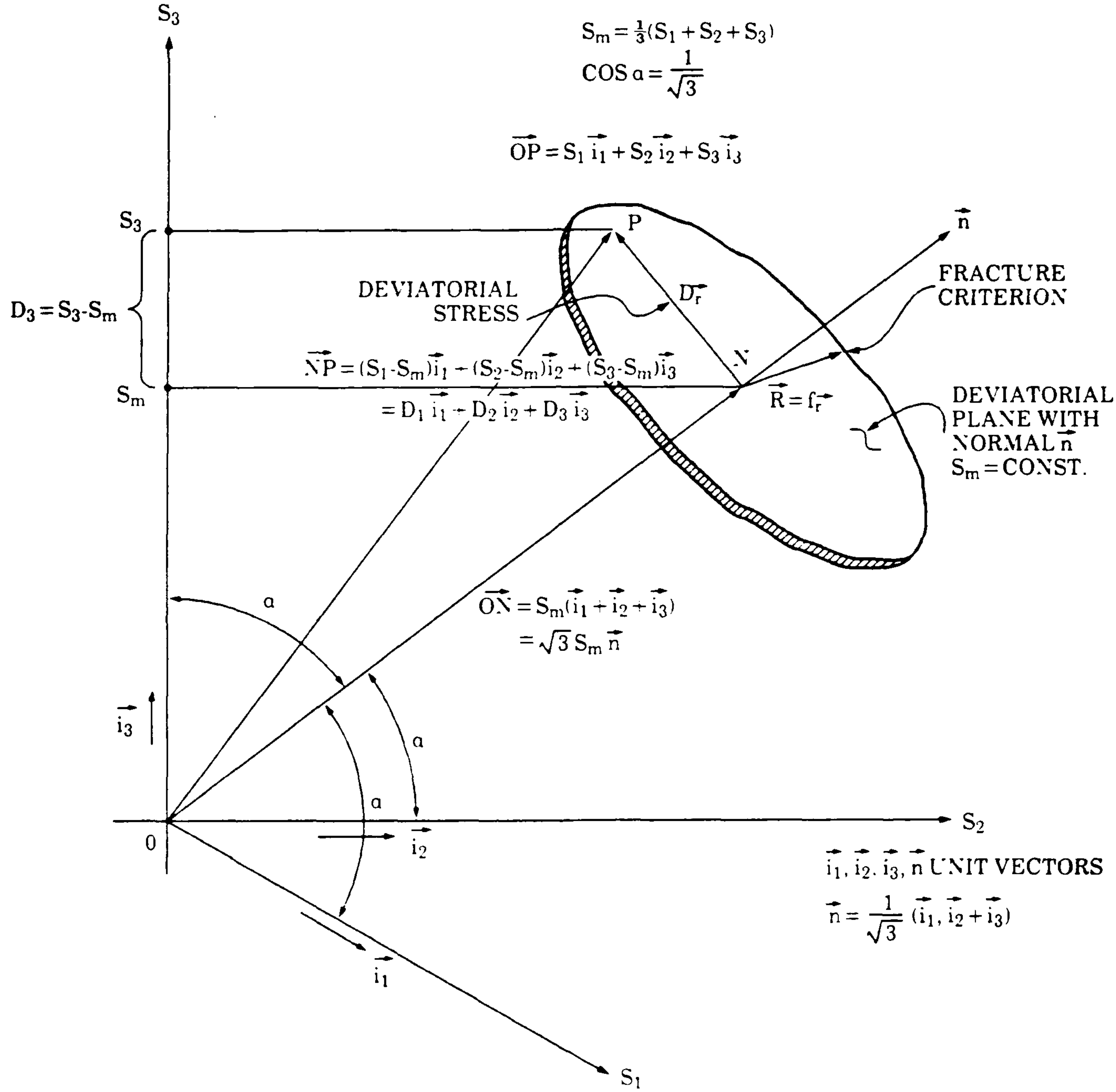

FIGURE 1. GEOMETRICAL REPRESENTATION OF STRESSES IN PRINCIPAL STRESS SPACE

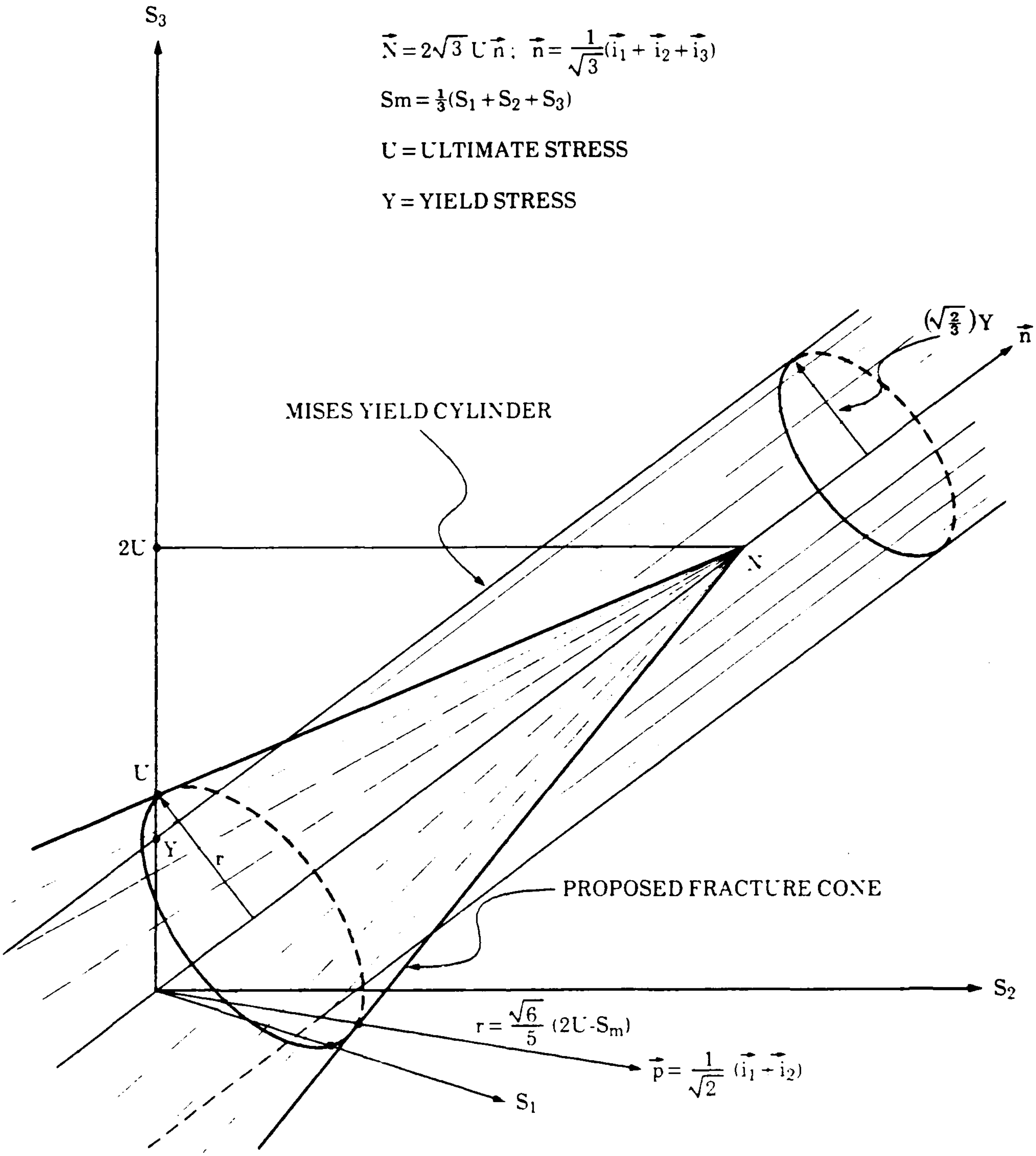

FIGURE 2. GEOMETRICAL REPRESENTATION OF FRACTURE AND YIELD CRITERIA

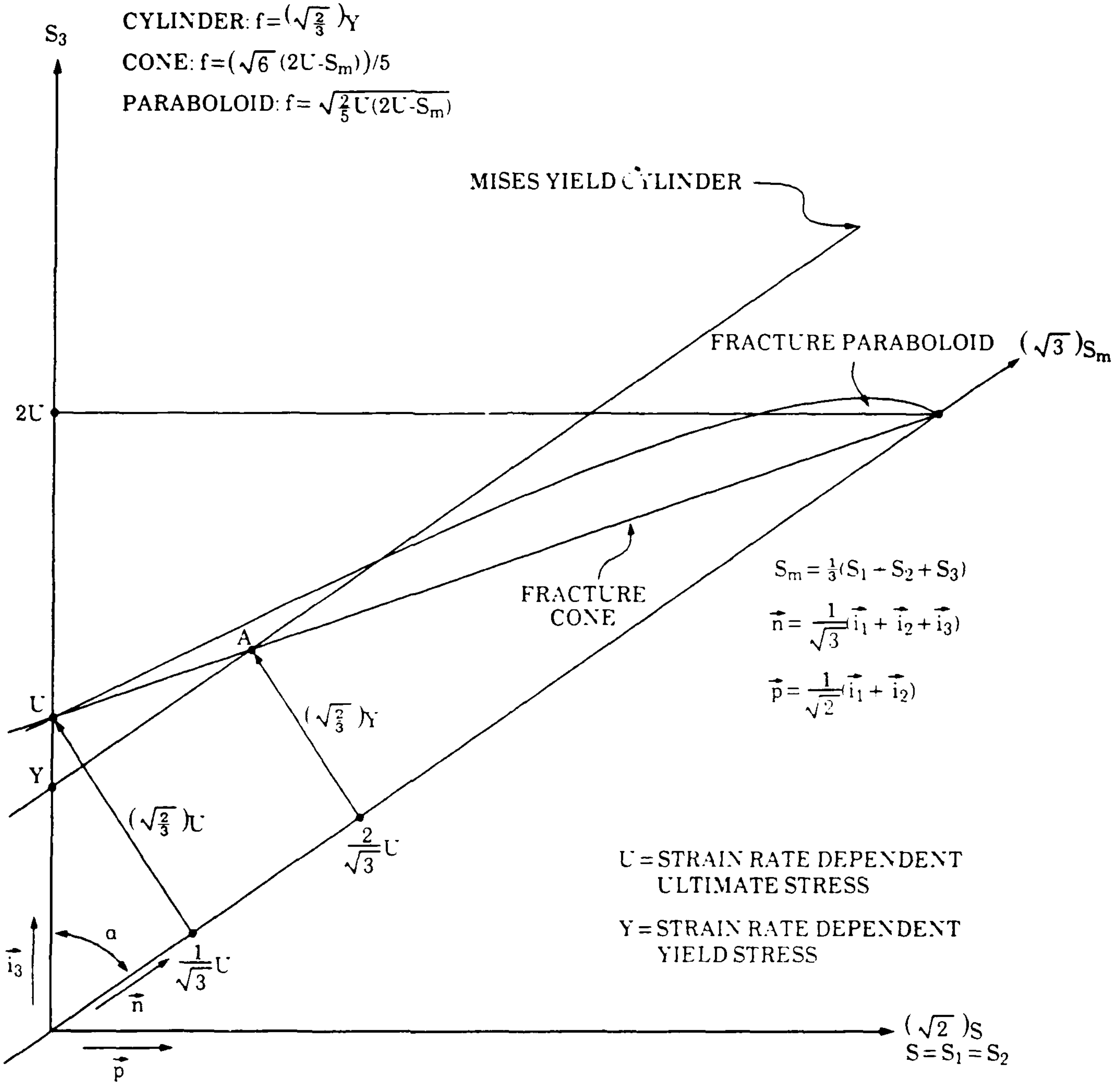

FIGURE 3. REPRESENTATIONS OF CRITERIA IN S_3-S_m PLANE

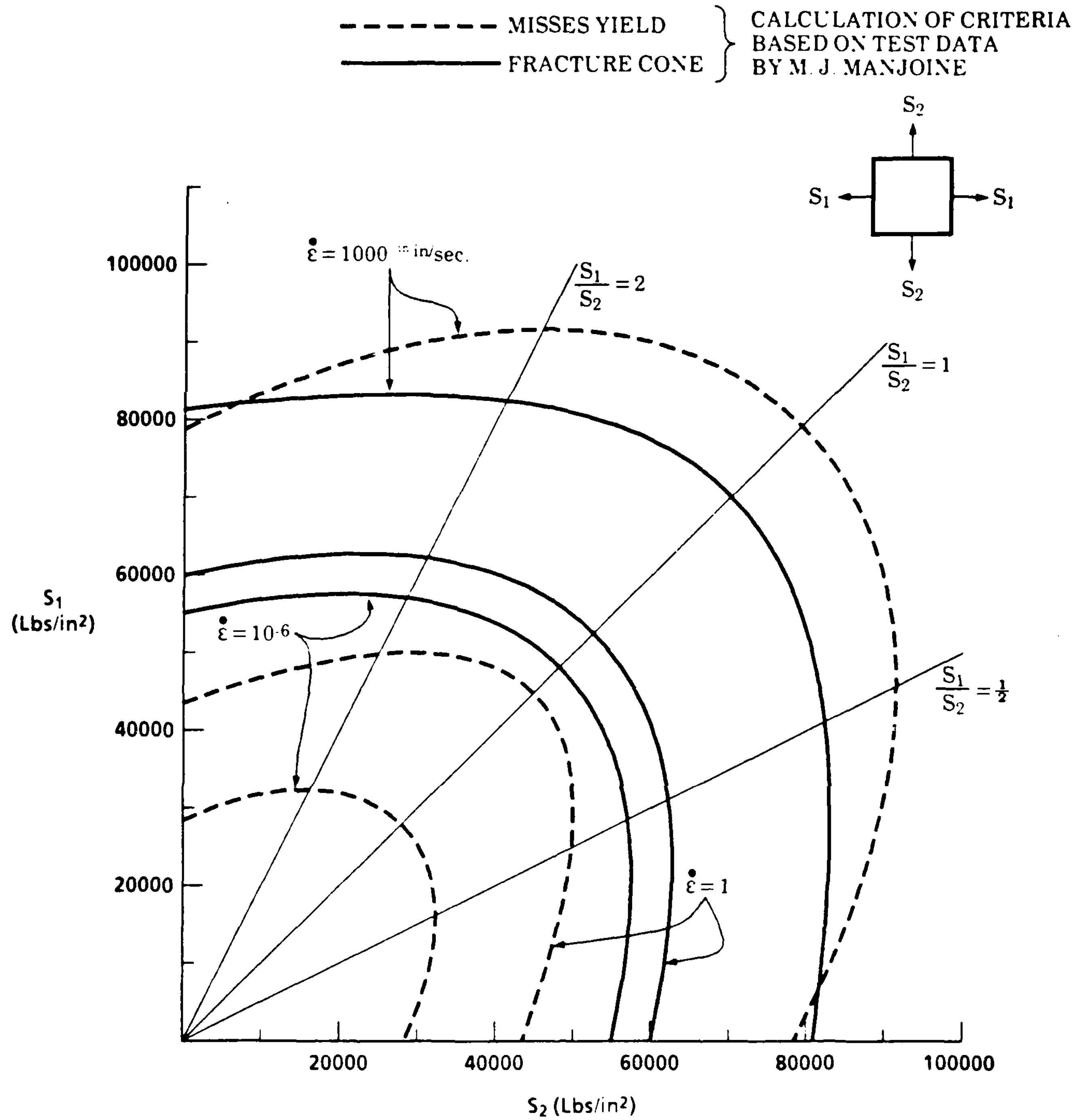

FIGURE 4. PLANE STRESS YIELD AND FRACTURE CRITERIA OF MILD STEEL AT VARIOUS STRAIN RATES

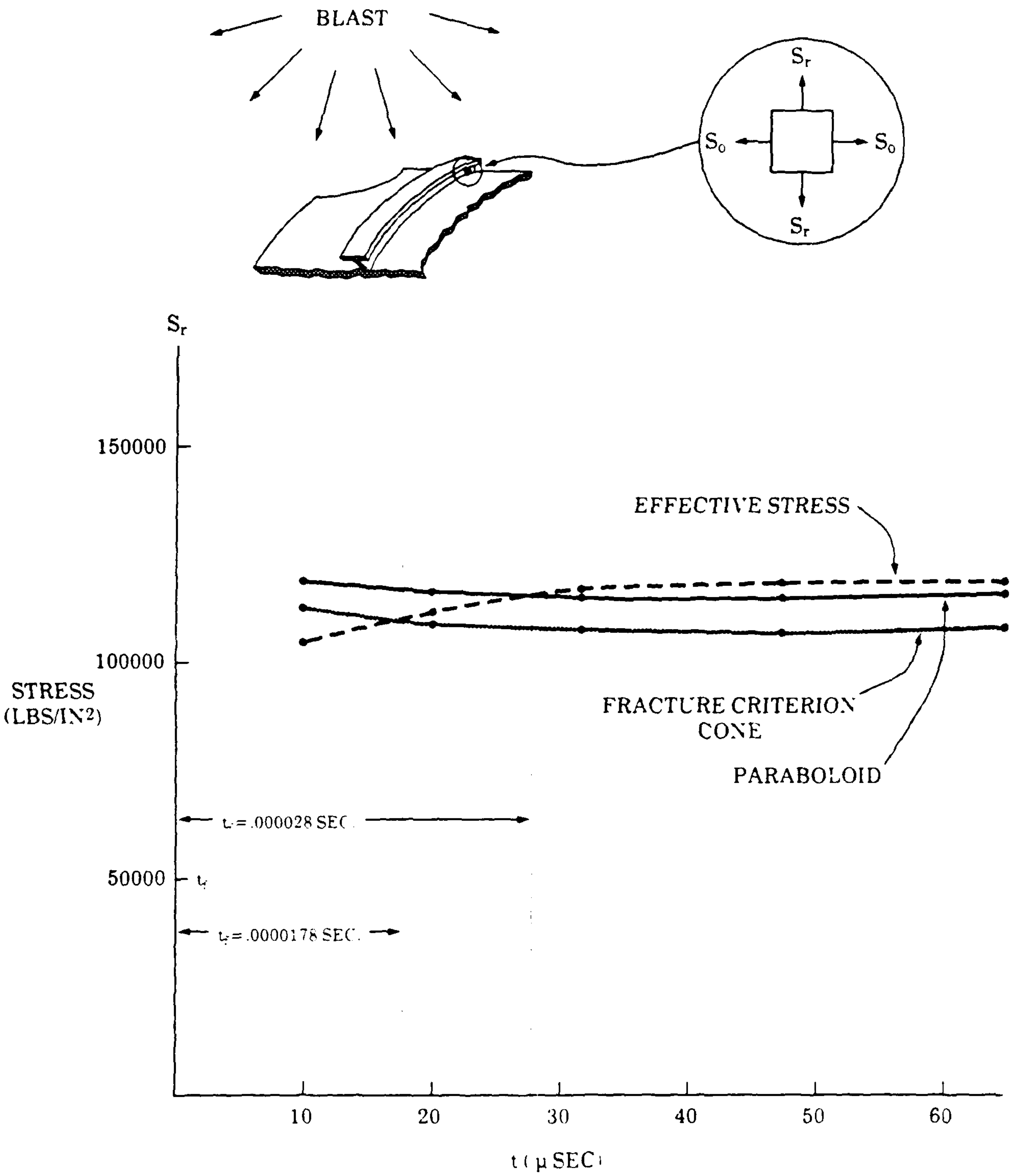

FIGURE 5 FRACTURE ANALYSIS OF "G" MODEL

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